CS 188: Artificial Intelligence
Spring 2010

Lecture 24: Perceptrons and More!
4/20/2010

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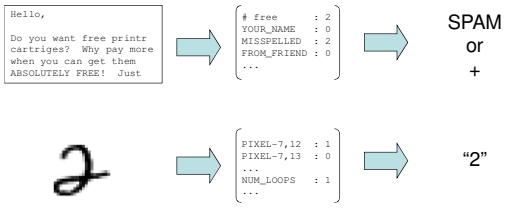
Slides adapted from Dan Klein

Announcements

- W7 due Thursday [that's your last written for the semester!]
- Project 5 out Thursday
- Contest running

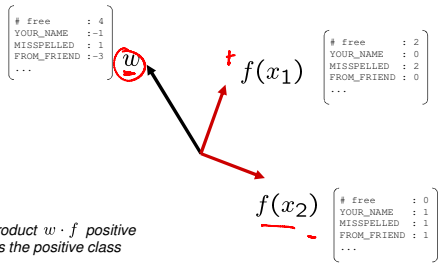
Classification: Feature Vectors

x $f(x)$ y



Classification: Weights

- Binary case: compare features to a weight vector
- Learning: figure out the weight vector from examples



Dot product $w \cdot f$ positive means the positive class

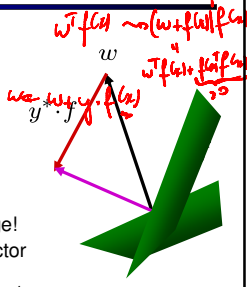
Learning: Binary Perceptron

- Start with weights = 0
- For each training instance:
  - Classify with current weights

$$y = \begin{cases} +1 & \text{if } w \cdot f(x) \geq 0 \\ -1 & \text{if } w \cdot f(x) < 0 \end{cases}$$

  - If correct (i.e., y=y*), no change!
  - If wrong: adjust the weight vector by adding or subtracting the feature vector. Subtract if y* is -1.

$$w = w + y^* \cdot f$$



[Demo]
http://isl.ira.uka.de/neuralNetCourse/2004/VL_11_5/Perceptron.html

Multiclass Decision Rule

- If we have multiple classes:
  - A weight vector for each class:

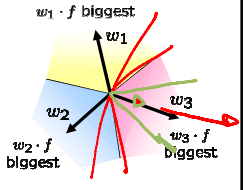
$$w_y$$

  - Score (activation) of a class y:

$$w_y \cdot f(x)$$

  - Prediction highest score wins

$$y = \arg\max_y \; w_y \cdot f(x)$$



Binary = multiclass where the negative class has weight zero

Learning: Multiclass Perceptron

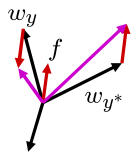
- Start with all weights = 0
- Pick up training examples one by one
- Predict with current weights

$$y = \arg\max_y \; w_y \cdot f(x)$$

- If correct, no change!
- If wrong: lower score of wrong answer, raise score of right answer

$$w_y = w_y - f(x)$$

$$w_{y^*} = w_{y^*} + f(x)$$



Example: Multiclass Perceptron

"win the vote"

"win the election"

"win the game"

w_{SPORTS}

BIAS	: 1
win	: 0
game	: 0
vote	: 0
the	: 0
...	

$w_{POLITICS}$

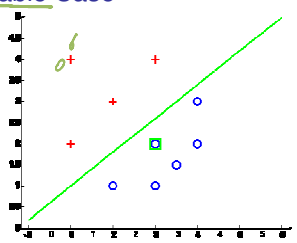
BIAS	: 0
win	: 0
game	: 0
vote	: 0
the	: 0
...	

w_{TECH}

BIAS	: 0
win	: 0
game	: 0
vote	: 0
the	: 0
...	

Examples: Perceptron

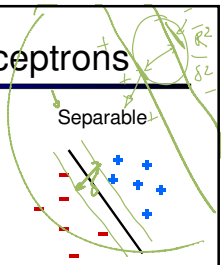
- Separable Case

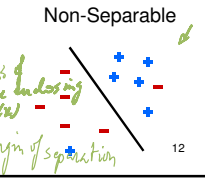


Properties of Perceptrons

- Separability: some parameters get the training set perfectly correct
- Convergence: if the training is separable, perceptron will eventually converge (binary case)
- Mistake Bound: the maximum number of mistakes (binary case) related to the *margin* or degree of separability

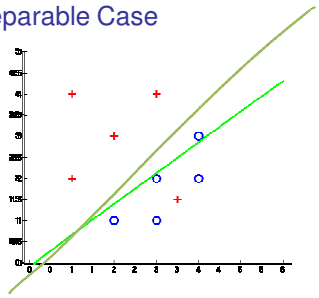
$$\text{mistakes} < \frac{R^2}{\delta^2}$$





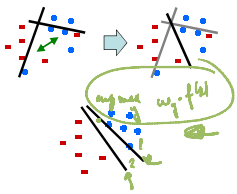
Examples: Perceptron

- Non-Separable Case



Problems with the Perceptron

- Noise: if the data isn't separable, weights might thrash
  - Averaging weight vectors over time can help (averaged perceptron)
- Mediocre generalization: finds a "barely" separating solution
- Overtraining: test / held-out accuracy usually rises, then falls
  - Overtraining is a kind of overfitting



Fixing the Perceptron

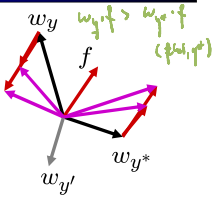
- Idea: adjust the weight update to mitigate these effects
- MIRA*: choose an update size that fixes the current mistake…
- … but, minimizes the change to w

$$\min_w \frac{1}{2}\sum_y ||w_y - w'_y||^2$$

$$w_{y^*} \cdot f(x) \geq w_y \cdot f(x) + 1$$

- The +1 helps to generalize

* Margin Infused Relaxed Algorithm



Guessed y instead of y^* on example x with features $f(x)$

$$w_y = w'_y - \tau f(x)$$
$$w_{y^*} = w'_{y^*} + \tau f(x)$$

Minimum Correcting Update

$$\min_w \frac{1}{2}\sum_y ||w_y - w'_y||^2$$
$$w_{y^*} \cdot f \geq w_y \cdot f + 1$$

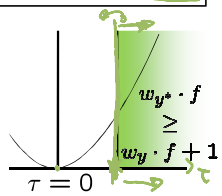
$$\min_\tau ||\tau f||^2$$
$$w_{y^*} \cdot f \geq w_y \cdot f + 1$$

(1)

$$(w'_{y^*} + \tau f) \cdot f = (w'_y - \tau f) \cdot f + 1$$
$$\tau = \frac{(w'_y - w'_{y^*}) \cdot f + 1}{2f \cdot f}$$

$$w_y = w'_y - \tau f(x)$$
$$w_{y^*} = w'_{y^*} + \tau f(x)$$

(1)



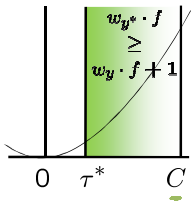
min not τ=0, or would not have made an error, so min will be where equality holds

Maximum Step Size

- In practice, it's also bad to make updates that are too large
  - Example may be labeled incorrectly
  - You may not have enough features
  - Solution: cap the maximum possible value of τ with some constant C

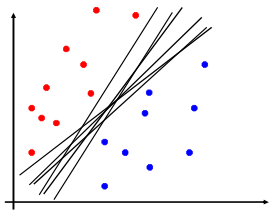
$$\tau^* = \min\left(\frac{(w'_y - w'_{y^*}) \cdot f + 1}{2f \cdot f}, C\right)$$

  - Corresponds to an optimization that assumes non-separable data
  - Usually converges faster than perceptron
  - Usually better, especially on noisy data



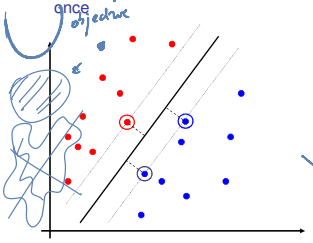
Linear Separators

- Which of these linear separators is optimal?



Support Vector Machines

- Maximizing the margin: good according to intuition, theory, practice
- Only support vectors matter; other training examples are ignorable
- Support vector machines (SVMs) find the separator with max margin
- Basically, SVMs are MIRA where you optimize over all examples at once



MIRA

$$\min_w \frac{1}{2}||w - w'||^2$$
$$w_{y^*} \cdot f(x_i) \geq w_y \cdot f(x_i) + 1$$

SVM

$$\min_w \frac{1}{2}||w||^2$$
$$\forall i, y \; w_{y^*} \cdot f(x_i) \geq w_y \cdot f(x_i) + 1$$

Separable case

hard margin SVM:

$$\min_w \frac{1}{2}||w||_2^2$$
$$\text{s.t. } \forall i, \forall y \neq y_i^* \quad w_{y_i^*} \cdot f(x_i) \geq w_y \cdot f(x_i) + 1$$

Inseparable case

soft margin SVM:

$$\min_{w,\xi} \frac{1}{2}||w||_2^2 + C\sum_i \xi_i$$
$$\text{s.t. } \forall i, \forall y \neq y_i^* \quad w_{y_i^*} \cdot f(x_i) \geq w_y \cdot f(x_i) + 1 - \xi_i$$
$$\forall i \quad \xi_i \geq 0$$

slack variable

Classification: Comparison

- Naïve Bayes
  - Builds a model training data
  - Gives prediction probabilities
  - Strong assumptions about feature independence
  - One pass through data (counting)
- Perceptrons / MIRA / SVM:
  - Makes less assumptions about data
  - Mistake-driven learning
  - Multiple passes through data (prediction)
  - Often more accurate

Extension: Web Search

- Information retrieval:
  - Given information needs, produce information
  - Includes, e.g. web search, question answering, and classic IR
- Web search: not exactly classification, but rather ranking

x = "Apple Computers"

Feature-Based Ranking

x = "Apple Computers"

$f($  $) = [0.3\ 5\ 0\ 0\ \ldots]$

$f($  $) = [0.8\ 4\ 2\ 1\ \ldots]$

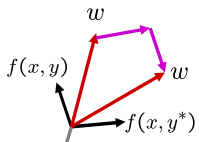
Perceptron for Ranking

- Inputs x
- Candidates y = web page
- Many feature vectors: $f(x,y)$
- One weight vector: w
  - Prediction:

$$y = \arg\max_y \ w \cdot f(x,y)$$

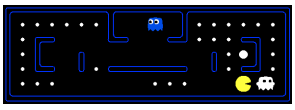
  - Update (if wrong):

$$w = w + f(x,y^*) - f(x,y)$$



Pacman Apprenticeship!

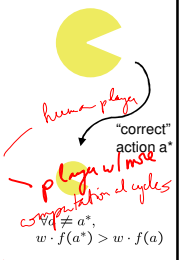
- Examples are states s



- Candidates are pairs (s,a)
- "Correct" actions: those taken by expert
- Features defined over (s,a) pairs: f(s,a)
- Score of a q-state (s,a) given by:

$$w \cdot f(s,a)$$

- How is this VERY different from reinforcement learning?



$$\text{if } a \neq a^*, \quad w \cdot f(a^*) > w \cdot f(a)$$